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Integral Relations for Disturbance Isolation

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Introduction

PASSIVE and active dynamic systems of high order are employed for the purpose of disturbance isolation. Such systems' theory and design methods could benefit from using the design theory of active electrical networks.[1] The feasibility and the available performance of such systems can be evaluated using Bode integrals. This enables the system engineer to resolve the design tradeoffs without actually designing the system and the subsystems. A Bode integral is applied to estimate the performance of a disturbance isolation system.

Consider the system in Fig. 1a of two bodies connected with an active strut,[2] which is a linear motor. A force disturbance source F_1 is applied to the body M_1 (capital letters designate Laplace transforms). The force F_3 is applied via the massless active strut to the body M_3. To increase the disturbance isolation, the force division ratio $K_F = F_3/F_1$ should be made small. The strut mobility (in some literature called mechanical impedance) Z_2 is the ratio of the difference in the velocities at the ends of the strut to the force (because we neglect the strut's mass, the force is same at the both ends of the strut). Feedback is employed in the active strut to increase $|Z_2|$ in order to reduce $|K_F|$.

For the purpose of analysis we use the following electromechanical analogy: power to power, voltage to velocity, current to force, capacitance to mass, and inductance to the inverse of the stiffness coefficient. The electrical equivalent circuit for the system is shown in Fig. 1b. The current division ratio I_3/I_1 is equivalent to K_F. The electrical impedance Z_2 is the equivalent of the strut mobility.

The current division ratio, i.e., the force division ratio, is

$$K_F = \frac{1/(sM_1)}{1/(sM_1) + Z_2 + 1/(sM_3)}$$

or

$$K_F = \frac{1}{1 + sM_1Z_2 + M_1/M_3} \tag{1}$$

At higher frequencies, the strut equivalent electrical impedance in Fig. 1b degenerates into the impedance of the series inductance included in Z_2, the inductance being equivalent to the inverse of the stiffness coefficient k of the strut at higher frequencies. Therefore, the force division ratio at higher frequencies turns into

$$K_F|_{\omega\to\infty} = k/s^2M_1 \tag{2}$$

Because this value reduces as a square of the frequency, Bode integral of the real part of a function[1,3] applies. From this integral,

$$\int_0^\infty \log|K_F + 1|\,d\omega = 0 \tag{3}$$

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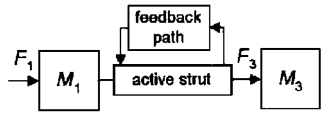
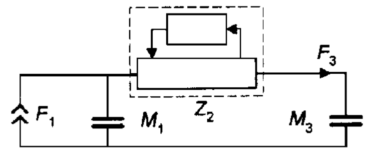


a) Mechanical system



b) Electrical equivalent circuit

Fig. 1 Two-body system.

This relation remains valid with and without feedback in the active strut and allows one to estimate the effect of feedback on the disturbance isolation at higher frequencies, but only as long as the Fig. 1 model correctly reflects the physical strut and the bodies. The model must be accurate enough over the frequency range where $\log|K_F + 1|$ is substantial. The model might become inaccurate at higher frequencies within this range because the mass of the strut cannot be neglected at high frequencies and the bodies cannot be considered rigid. Integral relations can be developed for such systems as well[1]; however, the increased complexity of the integrand makes it more difficult to use the relations for fast performance estimation while comparing different design versions.

Another equation, which will give a better estimation of the available performance at lower frequencies, can be found as follows. From Eq. (1),

$$\frac{1}{K_F(1 + M_1/M_3)} = 1 + \frac{sM_1Z_2}{1 + M_1/M_3} \tag{4}$$

Consider the practical case of the feedback in the active strut to be finite at dc. At lower frequencies, the active strut degenerates into some spring. Therefore, at lower frequencies the fraction in the right-hand side of Eq. (4) increases with frequency as ω^2. With the frequency scale inverted, the fraction decreases at high frequencies as ω^{-2}. Then the Bode integral of the real part of a function applies, and from the integral, the integral of the logarithm of the expression in the right side of Eq. (4) equals 0. Therefore,

$$\int_0^\infty \left[-\log|K_F| - \log\frac{1 + M_1}{M_3}\right] d\omega^{-1} = 0$$

or

$$\int_0^\infty \log|K_F|\,d\omega^{-1} = -\int_0^\infty \log\frac{1 + M_1}{M_3}\,d\omega^{-1} \tag{5}$$

The feedback in the active strut does not affect the right-hand side of the equation. Therefore, when comparing the cases with different values of feedback in the active strut loops, the right-hand part of Eq. (5) can be neglected. Hence, the integral of the difference in the vibration transmission between any two cases with different feedback in the active strut is zero:

$$\int_0^\infty \Delta \log|K_F|\,d\omega^{-1} = 0 \tag{6}$$

Equation (6) is important because it places a simple fundamental restriction on what can be achieved by disturbance isolation design. Introduction of feedback in the active strut reduces the force division ratio at some frequencies, but at some other frequencies (in fact, at lower frequencies) this ratio increases, and the difference in the areas of the output force reduction and the force increase, with inverse frequency scale, is zero.

In experiments with a large-scale model of an interstellar interferometer, a vibration source (representing a reaction wheel) was placed on a platform (body 1) suspended on six orthogonal active struts. Vibration propagation to the base on which sensitive optics was installed (body 2) was reduced by the bandpass feedback in the active struts by 30 dB at 20 Hz, the value gradually decreasing with frequency to nearly zero at 100 Hz. At 7-Hz frequency, however, the feedback increased the base vibrations by approximately 10 dB. This tradeoff between the vibration amplification and vibration attenuation over different frequency ranges was quite acceptable since at lower frequencies, the feedback in the optical pointing loop was large (and can be made even larger, if necessary, by application of nonlinear dynamic compensation,[3] and the total error in the optical loops was small.

Conclusion

The relation (6) is convenient for estimation of the effects of disturbance isolation loops in complex feedback systems.

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Terminal Area Navigation Using a Relative Global Positioning System Correction Vector Scheme

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Introduction

THE purpose of differential or relative global positioning system (GPS) schemes is to provide affordable, all-weather precision strike capability for a guided weapon without employing a terminal guidance seeker. Precision strike capability is typically understood to imply an accuracy at the target on the order of 3-m circular error probable (CEP), which is substantially better than the accuracy associated with the standard GPS precise positioning service level of 16-m spherical error probable. Differential or relative GPS schemes are currently receiving much attention because they offer the potential of meeting the U.S. Department of Defense's identification of all-weather, precision strike weaponry as a focus of modern weapon systems development. In general, the differential or relative GPS schemes proposed to date involve the use of two GPS receivers, one in the guided weapon and a second cooperative receiver, with communication between the two.[1]

The basic differential GPS scheme utilizes a cooperative GPS receiver located at a precisely known position, referred to as the base station. The purpose of the GPS receiver is to provide an estimate of the random biaslike pseudorange errors present in the pseudorange measurements for each of the visible satellites, and then to transmit these to a weapon equipped with a GPS receiver that is operating in the vicinity. The extent to which these errors are eliminated from the weapon's position solution depends on the degree of spatial and temporal correlation that these errors exhibit between the weapon and the base station. The drawback to this scheme is that the correlations degrade substantially over the times and distances typical of most tactical engagement scenarios, thus rendering the pseudorange random bias cancellation not effective. If the cooperative GPS receiver is located onboard a mobile targeting platform such as an airborne warning and control system, then the precision strike scheme is more properly referred to as relative GPS. In this implementation, the relative location of the target with respect to the targeting platform is known very accurately, e.g., through the use of the targeting aircraft's synthetic aperature radar. If the precision strike weapon's GPS receiver can maintain a high degree of correlation of its GPS position solution bias errors with those of the targeting platform, the accuracy achieved at the target can be significantly better than that of absolute GPS.[2] Note that the accuracy claims for relative GPS navigation have been verified, for separations in time and horizontal displacement, through experiments involving the placement of GPS receivers at various sites across the United States.[3]

In the relative GPS correction vector scheme considered, a high-quality terrain scene is used in place of the cooperative GPS receiver to achieve precision strike capability. The location of the terrain scene relative to the target is known very precisely, a fact that is guaranteed by the appearance of both on the same satellite photograph (geocell). At the time of overflying the terrain scene, a three-dimensional correction vector is formed in the navigation software of the precision strike weapon. This vector, which represents the difference between the estimated weapon position and the center of the terrain scene, is then applied outside of the system Kalman filter to correct all GPS positional solutions following the terrain scene. Application of the correction vector has the effect of essentially eliminating the large pseudorange bias errors inasmuch as GPS updates following the terrain scene are highly correlated with that at the terrain scene given reasonable scene/target separations. Furthermore, the weapon navigates in the terrain scene/target relative coordinate system where the absolute target errors are not important.

Correction Vector Implementation

To illustrate pictorially the performance of the relative GPS correction vector concept, the dynamics that occur in the terminal area are shown in Fig. 1. The weapon follows an actual trajectory that differs from the planned trajectory (as determined by mission planning) by the navigation position error. The primary components of the navigation position error are the pseudorange bias errors associated with the pseudorange measurements to each of the satellites being tracked. In the absence of navigation error, the planned trajectory would take the weapon to the planned world geodetic system-(WGS-) 84 coordinates of the scene, which differ from the actual scene coordinates by the large absolute scene location error. The planned WGS-84 target coordinates also differ from the actual target coordinates by the large absolute target location error, which is nearly equal to the absolute scene location error, assuming the terrain scene and target are on the same satellite photograph. Application of the correction vector at the location of the terrain scene restores the estimated trajectory (dashed line) to within a small distance of the actual scene center, i.e., the scene center error. This error arises from the inability to precisely locate the scene center, and is related to the size of the digitized terrain scene cells. The estimated trajectory will now follow a new path to the planned target. The actual trajectory will then follow a path parallel to the new path continually incorporating the correction vector. This results in the weapon arriving at the indicated impact point. This point differs from the actual target location primarily by the sum of the scene center and scene/target relative errors, which is significantly less than the sum of the pseudorange and absolute target (or scene) location random bias errors.

Application of the correction vector to GPS updates is performed external to the Kalman filter, thus resulting in no modification to the structure of the onboard filter. The corrected position solution

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